I. Introduction and Motivation

What is and where can we expect *magnetism* in QCD?

(I) Phase diagram of QCD on the $T-\mu$ plane
Where can we expect magnetism

Plion Condensation

\[ \langle \pi^0 \rangle = A \cos k_c z \]

Dual chiral density wave

\[ \langle \sigma_{12} \rangle = M \cos(\mathbf{q} \cdot \mathbf{r}) \]
(II) Strong magnetic field and evolution of compact stars

Its origin has been a long-standing problem

e.g. $O(10^{15}\text{ G})$ for magnetars

Young but rotating slowly

…

(cf. Makishima-san’s talk)

Origin:
(i) Fossil field
(ii) Dynamo scenario (crust)
(iii) Microscopic origin (core)
II. Ferromagnetic transition

A perturbative calculation based on

*Bloch mechanism* (repulsive exchange int. + Pauli pr.)

$$\Delta \varepsilon \equiv \varepsilon_{\text{ferro}} - \varepsilon_{\text{para}}$$

$$n_B^c \sim O(n_{\text{nuclear}})$$

$$n_q = 0.2 \text{ (fm}^{-3}\text{)}$$

$$n_q = 0.1 \text{ (fm}^{-3}\text{)}$$

$$n_{\pm} = (1 \pm p)n/2$$

$$B_{\text{max}} = \frac{8\pi}{3} \left( \frac{r_Q}{R} \right)^3 \mu_Q n_Q$$

$$\approx O(10^{15-17} \text{ G})$$

$$n_Q = O(0.1 \text{ fm}^{-3})$$
Magnetic susceptibility within Fermi liquid theory

\[ q = |p - k| = 0 \]
\[ \Delta k_F \]
\[ B \rightarrow 0 \]

spin susceptibility

\[ \chi_M = \frac{\partial \langle M \rangle}{\partial B} \bigg|_{N,T,B=0} \]
\[ \chi_M \rightarrow \infty \quad \text{or} \quad \chi_M^{-1} \rightarrow 0 \]

for spontaneous magnetization (ferromagnetism)

\[ \chi_M = \left( \frac{g_D \mu_q}{2} \right)^2 N(T) \left( 1 + N(T) \bar{f}^o \right) \]
\[ \left( \frac{g_D \mu_q}{2} \right)^2 \left( \frac{\pi^2}{N_C k_F E_F} - \frac{1}{3} f_1^s + \bar{f}^a \right) \]

Quasiparticle interaction:

\[ f_{k \zeta, q \zeta'} = f_{kq}^s + \zeta \zeta' f_{kq}^a \]

Spin dep.
\[ \zeta = \pm 1 \]

- Infrared (IR) divergences in QCD/QED
- \( f_1^s, \bar{f}^a \propto m \)

Heavy quarks are favored
Screening effects in QCD/QED

\( \chi_{OGE} \rightarrow \chi_{\text{screen}} \)

\[
D_{\mu\nu}(p) = P_{\mu\nu}^T D_T(p) + P_{\mu\nu}^L D_L(p) - \xi \frac{p_\mu p_\nu}{p^4},
\]

\[
D_{T(L)}(p) = \left( p^2 - \Pi_{T(L)} \right)^{-1}
\]

\[
\Pi_L(p) = m_D^2 \equiv \sum_{f=u,d,s} \left( g^2 / 2\pi^2 \right) \mu_f k_F^f, \quad \text{(Debye mass)}
\]

\[
\Pi_T(p) = -i \frac{\pi u_F m_D^2}{4} \frac{p_0}{|p|}, \quad u_F \equiv k_F / E_F \quad \text{(Landau damping)}
\]

(i) Debye (static) screening for the longitudinal (electric) gluons improves IR behavior

(ii) Transverse (magnetic) gluons only receives the dynamic screening to leave IR divergences,

(a) divergences cancel each other to give a finite \( \chi \) at \( T=0 \)

(b) \( \chi \) exhibits Non-Fermi-liquid behavior at finite \( T \).
Spin susceptibility at $T=0$

$\Delta \chi^{-1} \propto \kappa \ln \left( \frac{2}{\kappa} \right)$

$\kappa = \frac{m_D^2}{2k_F^2}$

$m_D^2 = \sum_{\text{flavors}} \frac{g^2}{2\pi^2} k_{F,i} E_{F,i} \propto N_F$

Screening favors spontaneous magnetization at large $N_F$
Non-Fermi-liquid effect at finite temperature

- We consider the low T case, $T/\mu << 1$, but the usual low-T expansion cannot be applied.
- Quasiparticle energy exhibits an anomalous behavior near the Fermi surface

For $\omega \sim \mu$ ((C. Manuel, PRD 62(2000) 076009)

$$\text{Re} \Sigma_+ (\omega) \approx \text{Re} \Sigma_+ (\mu)$$

Effective coupling (infrared free)

$$C_f = \frac{N_c^2 - 1}{2N_c}$$

$$- \frac{C_f g^2 u_F}{12\pi^2} (\omega - \mu) \ln \frac{\Lambda}{|\omega - \mu|}$$

$$+ \Delta^{\text{reg}} (\omega - \mu),$$

$$v_F^{-1} \text{ or } z_+^{-1} \propto \lim_{\omega \to \mu} \frac{\partial \text{Re} \Sigma_+}{\partial \omega} \to \infty$$

Marginal Fermi liquid due to transverse gluons

(R.P. Smith et al., Nature 455 (2008) 1220.)
Magnetic susceptibility at $T>0$

$$\chi^{-1}(T) \sim \chi^{-1}(T = 0) + \frac{\pi^4}{6N_c k_F^5 E_F} \left( 2E_F^2 - m^2 + \frac{m^4}{E_F^2} \right) \left( T^2 - \frac{C_g N_c \nu_F}{3\pi^2} T^2 \ln \left( \frac{T}{M_D} \right) \right)$$

usual $T^2$ term

Non Fermi-liquid effect

cf. Specific heat

$C_V \approx T \ln T$  (A. Ipp et al., PRD 69(2004)011901)

Gap equation

$\Delta \approx \exp \left[ -\left( \pi^2 + 4 \right) \left( N_C - 1 \right) / 16 \right] \exp \left( -3\pi^2 / \sqrt{2g} \right)$

(D.T. Son, PRD 59(1999)094019)

Phase diagram

Curie (critical) temperature should be order of several tens (40-60) MeV.

III Magnetism and chiral symmetry (SDW)

Similarity to LOFF state in superconductor or vortex in superfluid

\[ \Delta \langle qq \rangle \neq 0 \text{ (uniform)} \iff \Delta=0 \text{ (normal)} \]

\[ \Delta(r) \text{ (LOFF)} \]

\[ \langle \bar{q}q \rangle \neq 0 \rightarrow \langle \bar{q}q \rangle = 0 \quad \text{(chiral transition)} \]

\[ \langle M \rangle = 0 \rightarrow \langle M \rangle \neq 0 \quad \text{(ferromagnetic transition)} \]

Dual chiral density wave (DCDW)

\[ \langle \bar{q}q \rangle = \Delta \cos(q \cdot r) \]

\[ \langle \bar{q}i\gamma^5 \tau_3 q \rangle = \Delta \sin(q \cdot r) \]

Spin(-flavor) density wave
Recently there are many works about non-uniform phases in the context of chiral transition: including chiral density wave (CDW) or real kink crystal (RKC), based on the large N argument and $Z_2$ symmetry.

CDW

$$\langle \bar{q}q \rangle = \sigma \cos(q \cdot r)$$

D.V. Deryagin et al., IJMPA7(1992)659.
R. Rapp et al., PRD63 (2001) 034008.

or RKC

$$\langle \bar{q}q \rangle = \lambda \left( \frac{2\sqrt{v}}{1+\sqrt{v}} \right) \text{sn} \left( \frac{2\lambda z}{1+\sqrt{v}} ; v \right)$$

D.Nickel, PRL 103(2009) 072301;
S. Carignano et al., arXiv:1007.1397.
DCDW as another non-uniform phase

A magnetic phase at moderate densities

Another restoration path in the $U(1)$ plane

**Dual Chiral Density Wave (DCDW):**

$$\langle \bar{q}q \rangle = \Delta \cos \theta$$

$$\langle \bar{q}i\gamma^5 \tau_3 q \rangle = \Delta \sin \theta$$

(U(1) symmetry)

$$M = 2G\Delta$$

$$\theta(=q \cdot r):$$

1 dim. "magnetic" order

Chiral-restoration path
\[ E^\pm(p) = \sqrt{E_p^2 + |q|^2 / 4 \pm \sqrt{(p \cdot q)^2 + M^2 |q|^2}} \]

Single particle spectrum for \( q / 2 > M \):

\[ E_{1,2} = \sqrt{p_z^2 + M^2} \pm \frac{|q|}{2} \quad (p_\perp = 0) \]

- Energy gain \( \Delta E \) due to the \( q \)-dependent DCDW-quark interaction

\[ \rightarrow \text{Analog of nesting (cf. CDW or SDW)} \]
\( \Omega_{\text{total}} \) (NJL model in the chiral limit)
Some results in the NJL model (chiral limit)

Order Parameters at $T=0$

Phase Diagram ($G\Lambda^2=6$)

Extension to include $m_c$ is straightforward:

$$\theta = qz \rightarrow -\frac{d^2\theta(z)}{dz^2} + m_{\pi}^* \sin \theta(z) = 0,$$

Deformed DCDW:

$$\langle \bar{\psi} \psi \rangle = -\Delta \ cn^2(m_{\pi}^* z / k, k) - sn^2(m_{\pi}^* z / k, k),$$

$$\langle \bar{\psi} i\gamma_5 \tau_3 \psi \rangle = -2\Delta sn(m_{\pi}^* z / k, k)cn(m_{\pi}^* z / k, k).$$
Peculiar features of DCDW:

(i) Symmetry

\[ T \otimes U(1) \rightarrow U_{\hat{p} + Q_5^3}(1) \]

(1+1 dim. analog of Skyrmion)

\[ \pi_1(U(1)) = \mathbb{Z} \]

NG boson (“phason”) has a hybrid nature of “pion (spin(-isospin) wave)” and “phonon”

(ii) Spin density wave (SDW) or “spiral magnet”

Magnetic moment:

\[ \langle \sigma_{12} \rangle = M \cos(q \cdot r) \]

\[ B_{\text{loc}} = O(10^{16} \text{ G}) \]

A kind of liquid crystal with two dim. ferromagnetic (FM) order + one dim. anti-ferromagnetic (AFM) order

IV. Summary and concluding remarks

1. Some magnetic properties of QCD have been discussed.

2. For a ferromagnetic transition, we have considered magnetic susceptibility $\chi(q=0)$ of QCD within Fermi-liquid theory. Roles of static and dynamic screening are figured out:
   - Static: $g^4 \ln g^{-2}$
   - Dynamic: $T^2 \ln T$

   Novel non-Fermi liquid effect!

3. For dual chiral density wave (DCDW), we have discussed the mechanism, features and its extension to include the symmetry breaking ($m_c \neq 0$).

4. Toward a unified description of magnetism in QCD:
   - Non-perturbative (instanton) effects
   - Chiral symmetry restoration at moderate densities

   e.g. NJL model as an effective model of QCD

   Fock exchange term gives another "magnetic" interaction:

   e.g. $G/8N_c \left( \overline{\psi} \sigma^{\mu \nu} \psi \right)^2 \rightarrow G / 4N_c \left( \overline{\psi} \Sigma \psi \right)^2$

   $\langle \overline{q} \Sigma q \rangle = 0$ or $\langle \overline{q} i \gamma^5 q \rangle = 0 \rightarrow \langle \overline{q} \Sigma q \rangle \neq 0$ or $\langle \overline{q} i \gamma^5 q \rangle \neq 0$
5. Observational signatures of magnetic phases in compact stars - thermal evolution as well as magnetic one?

- Roles of Nambu-Goldstone bosons (phasons or magnons) in the magnetic phases
- Dynamics of magnetic domain and vortex

\[ \text{e.g. } \beta \text{ decay} \]

6. Hadron-quark continuity?
GT resonance
Pionic nuclei

Modern NN pot.
G-matrix, Var. cals

PionCond.

Magnetic properties
Anti-Ferro. vs Ferro.

Deconfinement

DCDW

SSB of chiral symm.
Tensor correlations

Pion cooling

NG bosons
Spin wave
Consider the energy scale:

\[ H_{int} = \mu_i B, \quad \mu_i = e_i \hbar / 2m_i c \]

For \( B = 10^{15} \text{G} \),

<table>
<thead>
<tr>
<th>( m_i ) [MeV]</th>
<th>( E_{int} ) [MeV]</th>
<th>( E_{typ} )</th>
<th>( q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5 (-6)</td>
<td>2.5 (\times 10^{-3})</td>
<td>2.5 (\times 10^{-2}) (-2.5)</td>
</tr>
</tbody>
</table>

\[ E_{typ} \ll E_{int} \text{ (electrons)}, \quad E_{typ} > E_{int} \text{ (nucleons and quarks)} \]

Ferromagnetism or spin polarization

Nuclear matter calculations have shown negative results.

For recent references,
I.Bombaci et al, PLB 632(2006)638
G.H. Bordbar and M. Bigdeli, PRC76 (2007)035803

Spontaneous magnetization of quark matter
NJL model

\[ \mathcal{L}_{NJL} = \bar{\psi}(i\not{\partial} - m_c)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\tau_3\psi)^2] \]

Consider the chiral limit, \( m_c = 0 \). Under the mean-field approx.,

\[ \mathcal{L}_{MF} = \bar{\psi}i\not{\partial}\psi + 2G_1 [\langle \bar{\psi}\psi \rangle \bar{\psi}\psi + \langle \bar{\psi}\gamma_5\psi \rangle \bar{\psi}\gamma_5\psi] - G_1 [\langle \bar{\psi}\psi \rangle^2 + \langle \bar{\psi}\gamma_5\psi \rangle^2] \]

Weinberg transformation,

\[ \psi_W = \exp[i\gamma_5\tau_3 q \cdot r / 2]\psi, \]

to get the transformed Lagrangian within the Hartree approx.,

\[ \mathcal{L}_{MF}' = \bar{\psi}_W[i\not{\partial} - m - 1/2\gamma_5\tau_3q]\psi_W - G\Delta^2, \]

with \( m = -2G\Delta \) and \( q^\mu = (0, q) \)

Correspondence:

\[ \langle \bar{q}q \rangle \neq 0 \quad \iff \quad \langle \bar{q}_W q_W \rangle = \Delta(\neq 0) \]

\[ \langle \bar{q}_W i\gamma_5\tau_3 q \rangle \neq 0 \]

\[ \langle \bar{q}_W i\gamma_5\tau_3 q W \rangle = 0 \]

\[ q/2 \propto \nabla \theta \quad \text{(Axial - vector field)} \]

non-uniform \quad \quad \text{uniform}
(II) Strong magnetic field in compact stars

Its origin is a long-standing problem since the first discovery of pulsars.

Recent discovery of magnetars seems to revive this issue

Origin:
(i) Fossil field
(ii) Dynamo scenario (crust)
(iii) Microscopic origin (core)
Remarks:

(i) **Nesting** (Overhauser, Peiels) is important

Level crossing of the energy spectrum near the Fermi surface

\[ V \cos qx \]

\[ e^{ikx} \rightarrow e^{ikx} + e^{i(k \pm q)x} \]

\[ |q| = 2k_F \]

Model indep.

\[ \rightarrow \quad \text{SDW, CDW} \]
Overhauser mechanism

Level crossing

$$E^\pm(p) \rightarrow \sqrt{p_\perp^2 + (|p_z| \pm q/2)^2},$$
in the massless limit.

Thus, we can see that this mechanism is very similar to that of spin density wave by Overhauser.

$$\rightarrow q = O(2p_F)$$ due to the incomplete nesting.
(iii) Phason and spin-wave as NG modes in SDW

\[ \langle \sigma_{12} \rangle = M \cos(q \cdot r + \phi(r, t)) \]
Magnetization

\[ q = |\mathbf{p} - \mathbf{k}| = 0 \]

\[ \Delta k_F \]

\[ B \rightarrow 0 \]

Gordon identity \((\mathbf{A} = \mathbf{B} \times \mathbf{r} / 2)\):

\[
\int d^4 x L_{\text{int}}^{QED} = e_q \int d^4 x \bar{\psi} \gamma \cdot \mathbf{A} \psi = \left( \mu_q \right) \int d^4 x \bar{\psi} \left[ -i \mathbf{r} \times \nabla + \Sigma \right] \cdot \mathbf{B} \psi.
\]

\[
\langle \mathbf{L} \rangle = 0
\]

\[
\langle M \rangle \equiv \langle q \Sigma q \rangle = \frac{g_D \mu_q}{2} N_C \sum_k \left( \delta n_{k\uparrow} - \delta n_{k\downarrow} \right)
\]

\[
n_{k\xi} = \left\{ \exp \left[ \beta \left( \epsilon_k - \mu - 1/2 g_D \mu_q \xi B \right) \right] + 1 \right\}^{-1}
\]

with the gyromagnetic ratio \(g_D\).

Dirac magneton \(\mu_q\)

\[
\Delta N = N_C V k_F^2 \Delta k_F / 2 \pi^2
\]

Spin susceptibility

Change of the distribution function

\[
\delta n_{k\uparrow} - \delta n_{k\downarrow} = \frac{\partial n_k}{\partial \xi_k} \left[ -g_D \mu_q B + \delta \xi_{k\uparrow} - \delta \xi_{k\downarrow} \right]
\]

\[
\delta \xi_{k\xi} = N_C \sum_{\xi'} \int \frac{d^3 q}{(2\pi)^3} f_{k\xi, q\xi} \cdot \delta n_{q\xi}
\]
**Magnetic (spin) susceptibility in the Fermi liquid theory**

\[
\chi_M = \frac{\partial \langle M \rangle}{\partial B} \bigg|_{N,T,B=0}
\]

\[
\chi_M = \left( \frac{g_D \mu_q}{2} \right)^2 N(T) / \left( 1 + N(T) \bar{f}^o \right)
\]

\[
= \left( \frac{g_D \mu_q}{2} \right)^2 / \left( \frac{\pi^2}{N_C k_F E_F} - \frac{1}{3} f_1^s + \bar{f}^a \right)
\]

\( f \): Landau parameters

\( N(T) \): effective density of states at the Fermi surface

\[
N(T) \approx \frac{N_C k_F^2}{\pi^2} \left( \frac{\partial k}{\partial \epsilon_k} \right)_{k_F}
\]

\( \nu_F \equiv \frac{k_F}{\mu} - \frac{N_C k_F^2}{3\pi^2} f_1^s \) \quad \text{Fermi velocity}

which also measures the curvature of the free energy at the origin

Free energy

\[
F(\langle M \rangle) \quad \chi^{-1} < 0
\]

\[
F(\langle M \rangle) \quad \chi^{-1} > 0
\]

\( \chi \rightarrow \infty \) or \( \chi^{-1} \rightarrow 0 \)

spontaneous magnetization

Infrared (IR) singularities
Relativistic Fermi liquid theory (G. Baym and S. A. Chin, NPA262 (1976) 527.)

No direct int. Fock exchange int.

$$(\text{tr} \lambda_{\alpha} = 0)$$

$$\varepsilon(k\zeta; ai) = \delta E / \delta n(k\zeta; ai)$$

$$f_{k\zeta; ai, q\zeta'; bj} = \delta \varepsilon(k\zeta; ai) / \delta n(q\zeta'; bj)$$

No flavor dep. $$\Rightarrow f_{k\zeta; ai, q\zeta'; bj} = \delta_{ij} f_{k\zeta; a, q\zeta'b}$$

In the following we are concerned with only one flavor.

Color symmetric int.:

$$f^{s}_{k\zeta, q\zeta'} \equiv \frac{1}{N_c^2} \sum_{a,b} f_{k\zeta; a, q\zeta'; b} = \frac{m_q}{E_k} \frac{m_q}{E_q} M_{k\zeta, q\zeta'}$$

Ferromagnetism in gauge theories
V. Magnetic properties at $T=0$

Quasiparticle interaction:

$$f|_{|k|=|q|=k_F} = -\frac{N_C^2 - 1}{2N_C^2} g^2 \frac{m^2}{E_F^2} \left[ -M^{00} D_L(k-q) + M^{ii} D_T(k-q) \right]$$

Susceptibility

$$(\chi_M(T=0) / \chi_{\text{Pauli}})^{-1} = 1 - \frac{C_f g^2}{12\pi^2 E_F k_F} \left[ m(2E_F + m) \right]$$

$\chi_{\text{Pauli}}$: Pauli paramagnetism

$$\kappa = \frac{m_D^2}{2k_F^2}$$

Screening effect

$O(g^4 \ln g^{-2})$
\[ \chi_{\text{spin}} / \chi_{\text{free}} \]

\[ \chi_{\text{free}} = \frac{\mu k_F}{\pi^2} \]

Large fluctuations (or paramagnon)

\[ \Delta C_V \sim T^3 \ln T \] (Pethick and Carneiro)

Ferromagnetism

Paramagnetism

Note that this SDW has nothing to do with chiral symmetry, but nesting is also important.
Magnetic phase diagram of QCD

Curie (critical) temperature should be order of several tens of MeV.

Non-Fermi-liquid effect
VI. Finite temperature effects and Non-Fermi-liquid behavior

$$\chi_M = \left( \frac{g_D^F \mu_q}{2} \right)^2 \frac{N(T)}{1 + N(T) \left( f_l^a + f_t^a \right)} = \left( \frac{g_D^F \mu_q}{2} \right)^2 \frac{1}{N^{-1}(T) + f_l^a + f_t^a}$$

- Density of state:

$$N(T) = -2N_c \int \frac{d^3k}{(2\pi)^3} \frac{\partial n_k}{\partial \varepsilon_k}, \quad n_k = \frac{1}{1 + \exp(\beta(\varepsilon_k - \mu))}$$

- $f_l^a, f_t^a$: spin dependent FL interaction s.t.

$$f_i^a \equiv -2N_c \int \frac{d^3k}{(2\pi)^3} \frac{\partial n(\varepsilon_k)}{\partial \varepsilon_k} f_{i;k,k_s}^a / N(T) \left( \varepsilon_k = \mu \right)$$

- We consider the low T case, $T/\mu<<1$, but the usual low-T expansion cannot be applied.

- Quasiparticle energy exhibits an anomalous behavior near the Fermi surface
(ii) Similar idea

\[ \mathbb{Z}_2 \text{ (Ising)} \text{ vs } O(4) \]

Chiral density wave:

\[ \langle \bar{\psi} \psi \rangle = \sigma \cos(\mathbf{q} \cdot \mathbf{r}) \]

refs.


(iii) Similarity to LOFF in superconductor

\[ \Delta \text{ (uniform)} \leftrightarrow 0 \]

\[ \uparrow \]

\[ \Delta(r) \text{ (non-uniform)} \]
Magnetic aspects of QCD and compact stars

T. Tatsumi
Department of Physics, Kyoto University

I. Introduction and motivation
II. Chiral symmetry and spin density wave (SDW)
III. Ferromagnetism (FM) and magnetic susceptibility
IV. Screening effects for gluons
V. Magnetic properties at T=0
VI. Finite temperature effects and Non-Fermi-liquid behavior
VII. Summary and concluding remarks

Alternating Layer Spin [ALS] Structure

Figure 1: Alternating Layer Spin (ALS) structure associated with π and η phonons in superconducting BCS. Bold arrows show proton spin and thin arrows neutron spin.
II. Chiral symmetry and Spin density wave (SDW)

A typical example of the $\pi^0$ condensed phase:

$$\nabla \langle \psi^\dagger \sigma_3 \tau_3 \psi \rangle \propto \langle \pi^0 \rangle \propto \sin(k_c z)$$

Liquid crystal with antiferromagnetic order

---

**Alternating Layer Spin [ALS] Structure**

Figure 1: Alternating Layer Spin [ALS] structure associated with $\pi^0$ condensation (\$A_{\pi}\$) in symmetric nuclear matter. Bold arrows denote proton spins and thin arrows neutron spins.
I. Introduction and motivation

(1) Phase diagram of QCD

- Spin degrees of freedom
- Meson (Pion) condensation (PIC)
- Antiferromagnetic (AFM)
- Deconfinement
- Ferromagnetic (FM)
- Chiral restoration
- Spin density wave (SDW)
- Color superconductivity

Magnetic phase diagram of QCD

- Critical end-point
- Quark matter
- Hadron matter
- CSC
- PIC
- SDW
- FM

\( \rho_{\text{nuclear}} \quad ????? \quad \rho_B \)
II. Ferromagnetism (FM) and magnetic susceptibility

Is there ferromagnetic instability in QCD?

Fock exchange interaction is responsible to ferromagnetism in quark matter (Bloch mechanism)
c.f. Ferromagnetism of itinerant electrons (Bloch,1929)

\[ \alpha \lambda_{\alpha} \]

\[ \lambda_{\alpha} \]

\[ \left( \lambda_{\alpha} \right)_{ab} \left( \lambda_{\alpha} \right)_{ba} = 1/2 - 1/(2N_c) \delta_{ab} > 0 \]
Magnetism in QCD

Strong magnetic field in compact stars

Its origin is a long-standing problem

e.g. $O(10^{15} \text{ G})$ for magnetars
Role of transverse gluons = *relevant* interactions in RG

Effective coupling is $C_{\text{eff}} = g^2 v_F \rightarrow \text{Infrared free}$

(Schaefer, K. Schwenzer, PRD 70(2004) 054007)

Non-Fermi liquid behavior

- Specific heat $C_V \approx T \ln T$

(A. Ipp et al., PRD 69(2004) 011901)

- Gap equation $\Delta \approx \exp\left[-\left(\pi^2 + 4\right)(N_c - 1)/16\right] \exp\left(-3\pi^2 / \sqrt{2}g\right)$

(D.T. Son, PRD 59(1999)094019)

How about susceptibility?

Peculiar temperature dep. of susceptibility

$\rightarrow \text{Curie temperature}$
To summarize:

\[ \chi_{\text{Pauli}} = 1 - O(g^2) + O(g^4) \ln(g^{-2}) + \ldots \]

Some features:

(i) Debye screening in the longitudinal (electric) gluons improve IR behavior

(ii) Transverse (magnetic) gluons only gives the dynamical screening, which leads to IR (Log) divergence

\[ \rightarrow \quad \text{Non-Fermi-liquid behavior} \]

(iii) Divergences cancel each other to give a finite \( \chi \)

(iv) Results are independent of the gauge choice \( \xi \)